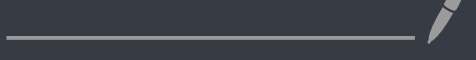

Session 3

Exercise 1

Prove that at $T=0$, the one dimensional Ising model has a spontaneous magnetization that depends on the limit for $h \to 0$ ($h \to 0^+$ and $h \to 0^-$)

Exercise 2

Solution of the 1D Ising model in the mean-field pair approximation, and use $F = \langle E \rangle + k_B T \langle \ln P \rangle$

Show that if we approximate

$$P(\{s_i\}) = \prod_{i=1}^{N-1} p(s_i, s_{i+1}) \cdot \prod_{i=1}^{N} p^{-1}(s_i) \qquad (1)$$

the solution correspond to the exact one!

a) How do you construct approx (1)?

Hint: use Bayes' rule applied to $P(\{s_i\})$

b) To find the exact solution, use

- $p(+1,-1) = p(-1,+1)$ (reflection symmetry)
- $p(+1,+1) + p(+1,-1) = p(+1)$
- $p(-1,+1) + p(-1,-1) = p(-1)$
- $p(+1) - p(-1) = m$
- $p(+1,+1) + p(-1,-1) - 2p(+1,-1) = c$
- $p(+1,+1) + p(-1,+1) + p(+1,-1) + p(-1,-1) = 1$